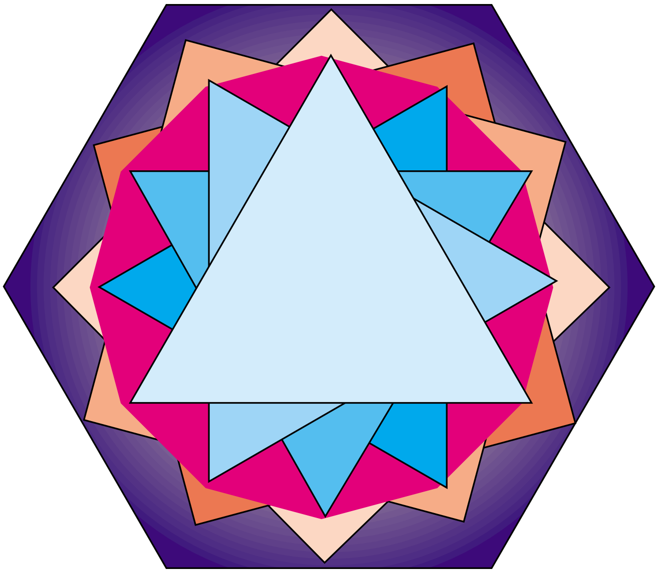
Maths Workbook 1

Number Patterns : 1



You should be able to work through this text by yourself. If you get stuck on an example, ASK ...; but do not ask until you have *thought* about the example.

DO NOT WRITE IN THIS BOOKLET

Copy the exercises into your Maths notebook

Introduction to "Number Patterns"

This is the first of a series of special workbooks written for pupils who needed extra material in the mixed ability class setting. This first book was issued in May 1978 for pupils in the First Forms at Trinity School, Carlisle, but was also used at times with other years when additional work was called for.

This (slightly revised) edition is offered to anyone who will find it of use. All comments welcome!

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NUMBER PATTERNS

Here is a set of numbers: 1, 4, 7, 10, 13 ...
What number should follow 13?
And what number should follow your last answer?
What do we do to each number to find the next one in the set?

Exercise A

Continue these sets, giving the next two numbers in the set:

a) 1, 2, 3, 4 ...
b) 1, 3, 5, 7 ...
c) 2, 4, 6, 8 ...

Are these sets formed in a similar way to the set {1, 4, 7, 10 ...} ?

What common rule can you find for all of them?

In this Workbook we are going to study patterns of this type.

Here are the answers to the questions so far.
In the pattern 1, 4, 7, 10, 13, ... the next two numbers are 16 and 19. We add 3 to each number to find the next number in the set.
Exercise A: answers
a) the next two are 5 and 6
b) the next two are 9 and 11
c) the next two are 10 and 12.

In each case we add a fixed amount to each number to get the next number in the set.

Exercise B

Look at each of the sets that follow; some of them are like those in Exercise A. Which of them are formed like those in Exercise A?

a) 1, 5, 9, 13, 17
b) 1, 2, 4, 8, 16
c) 9, 11, 13, 15
d) 8, 16, 24, 32
e) 1, 3, 6, 10, 15
f) 3, 7, 11, 15
g) 1 1½, 2, 2½, 3
h) 1, ½, ¼, ⅛,

Here are the answers to Exercise B

a) Yes. In each case a constant 4 is added.
b) No. the amounts added are different each time.
c) Yes. A constant 2 is added.
d) Yes. A constant 8 is added.
e) No.
f) Yes. A constant 4 is added.
g) Yes. A constant ½ is added.
h) No.

You may like to think about the way in which the other sets (b, e, h) are formed but we shall not look at these here but in later Workbooks.

This completes the introduction to the topic. In what follows you will not be given the answers to the exercises. Work through the booklet, copying the examples into your Maths notebook.

Number Patterns 1: The Arithmetic Progression

Here is a number pattern: 1, 3, 5, 7 ...; what number follows the 7? You may have recognised this set as the set of odd numbers; they obey a rule - the numbers increase by 2 each time and this helps you work out that the number after 7 is 9 and the one after that is 9 + 2 = 11.
We started the Workbook with the pattern 1, 4, 7, 10 ... and showed that we form the next number by adding 3 to a given number. The next two numbers in the set are 10 + 3 = 13 and 13 + 3 = 16.

A set of numbers like this, in which the numbers increase by a fixed amount each time, is called an ARITHMETIC PROGRESSION (AP for short).

In Exercise 1 add two more numbers to each set.

Exercise 1

1) 2, 4, 6, 8
2) 1, 8, 15, 22
3) 2, 3, 4, 5
4) 6, 10, 14, 18
5) 1, 1·5, 2, 2·5, 3

What about this pattern?
20, 18, 16, 14 ...
How do we get to each number from the preceding one?
This time the numbers go **down** by a fixed amount. Each time a constant amount has been subtracted; in this set 2 has been subtracted each time.

Exercise 1 continued

Add two more numbers to each set

6) 20, 16, 12, 8
7) 50, 40, 30, 20
8) 5·9, 5·6, 5·3, 5·0
9) 43, 40, 37

To summarise. The special feature of the AP is that the numbers increase (or decrease) by a constant amount.

"And how many hours a day did you do lessons?" said Alice, in a hurry to change the subject.

"Ten hours the first day," said the Mock Turtle, "nine the next, and so on."

"What a curious plan!" exclaimed Alice.

"That's the reason they're called lessons," the Gryphon remarked: "because they lessen from day to day."

This was quite a new idea to Alice, and she thought it over a little before she made her next remark. "Then the eleventh day must have been a holiday?"

"Of course it was," said the Mock Turtle.

"And how did you manage on the twelfth?" Alice went on eagerly.

"That's enough about lessons," the Gryphon interrupted in a very decided tone. "Tell her something about the games now."

(Any ideas what they did on the twelfth?)

An aside ...

The pattern {1, 3, 6, 10, 15 ..} is not an arithmetic progression because the difference between successive terms is not constant. The pattern 3, 6, 9, 10, 13, 16 ... is not an AP either, ... or is it? If I say it **is** an arithmetic progression you will say "Nonsense - the amounts added are not the same". But what if the numbers have been written in some base other than base ten? You might feel that this is cheating even though the pattern works perfectly well if it is in base twelve (think about it ...)

It is always a good idea to consider other bases when investigating patterns and problems in numbers; but you may take it for granted that from this point on the numbers in this book (unless we say otherwise) are all in base ten.

Back to where we were before Alice and her friends interrupted.
In an AP, for example: 11, 13, 15, 17 ...
we say the first **term** of the pattern is 11
the second **term** is 13
the third **term** is 15 and so on.
Each term is formed by adding 2 to the preceding term. This 2, the constant amount added, is called the **common difference**. (It is the *difference*, the "gap" between pairs of numbers in the set given and it is a fixed *common* amount.)

In the pattern 1, 4, 7, 10 ...
the first **term** is 1, the second **term** is 4 and the **common difference** is 3.

Exercise 2

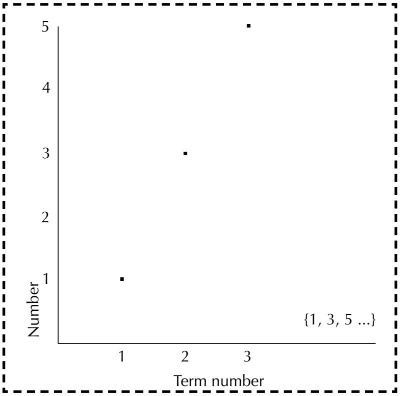
Give the first term and the common difference in
1) 2, 4, 6, 8
2) 3, 5, 7, 9
3) 5, 9, 13
4) 1, 8, 15, 22
5) 25, 24, 23, 22
6 8, 5, 2, -1

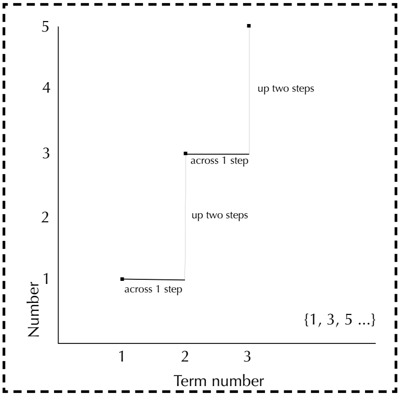
7) 2, 2¼, 2½, 2¾, 3

Note: the common difference can be positive or negative and does not have to be a whole number.

Dot Graphs

These APs can be shown as a dot graph or a step graph:





There is a connection between the dot graph and the graphs you draw for relations; if the dots were joined the graphs would form straight lines; but here, since we are dealing with terms of a progression - and there aren't any terms inbetween - joining the dots to form a line would have no meaning.

Exercise 3

Draw a dot graph for each of these sets

1) 2, 4, 6, 8
2) 1, 4, 7, 10
3) 8, 6, 4, 2
4) 0, 1, 2, 3

Looking at the AP

If we look at the way in which the AP is formed we can find ways of discovering any term in the set without writing them all out in order.

For example, in {1, 3, 5, 7, ...}

Term number	is	or	or
1	1	1	1
2	3	1 + 2	1 + 1(2)
3	5	1 + 2 + 2	1 + 2(2)
4	7	1 + 2 + 2 +2	1 + 3(2)

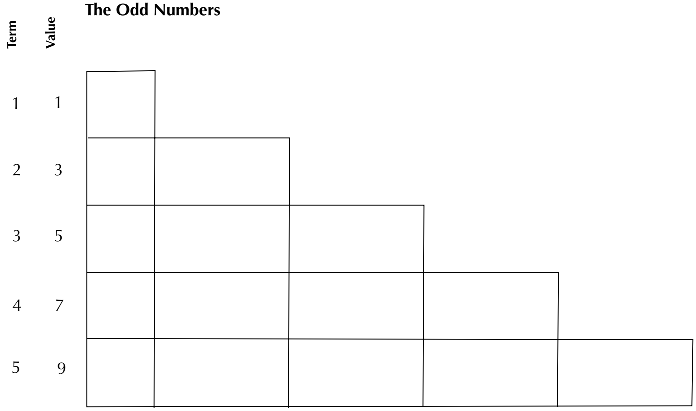
What's the next line?
By writing 1+ 2 + 2 + 2 as 1 + 3(2) in the fourth line for we show that we have added 3 lots of 2 to the first term, or 1 + 6 to get 7, the fourth term.
This shows another pattern.
To get to the second term (term ***2***) we add ***1*** difference
to get to term ***3*** we add ***2*** differences
to get to term ***4*** we add ***3*** differences
so to get to term ***5*** we add?

4 differences.

Term number	differences added
1	0
2	1
3	2
4	3
5	4

The number of differences added is 1 less than the term number

The Odd Numbers



9 = 1 + 2 + 2 + 2 + 2 = 1 + 4(2) = 1 + 8 = 9.

With this information we can now find any term without needing to list the members of the set as far as the term we want.

For example, the tenth term of the set of odd numbers is the sum of the first term (1) and (10 - 1) differences, i.e. 9 lots of 2. So the tenth term is 1 + 9(2) = 1 + 18 = 19.

What is the hundredth term of the AP 7, 12, ...?

The first term is 7 and we need (100-1) = 99 differences for the 100th term. The difference is 5, so the hundredth term is 7 + 99(5) or 7 + 495 = 502.

[NB in this example it's easier to work out 99 x 5 as (100-1)5 = 500 - 5 = 495 than by actually multiplying 99 by 5.]

Exercise 4

For each AP find the term asked for.

1) 2, 4, 6, 8, ... the seventh term
2) 3, 5, 7, 9, ... the sixth term
3) 1, 5, 9, 13, ... the tenth term
4) 25, 23, 21 ... the sixth term
5) 18, 8, -2, ... the fifth term
6) 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, ... the ninth term
7) 0·1, 0·3, 0·5, ... the twentieth term

Slightly harder questions:

8) The first term of an AP is -18 and the common difference is 7. How many of the terms of the AP (including the first) are negative? One term in the AP is 66; which one is it?

9) The first term of another AP is 15 and the common difference is -4. How many of the terms (including the first) are positive? One term in the AP is -49; which one is it?

> He thought he saw a butterfly
> That ate its weight in glue;
> He looked again, and found it was
> The cubic root of 2.
> If this is beauty, Lord, he sighed,
> I'll leave the Maths to You.

A GENERAL APPROACH

We can now show our patterns in a more general way by using algebra.

We will call the first term t_1, the second term t_2, the third term t_3 and so on, the number attached to the letter t telling us which term it is.

The letter d will stand for the common difference.

How is an AP built up?

We start with the first term

t_1 and add d to get t_2

$t_2 = t_1 + d$; add d again to get t_3

$t_3 = t_2 + d$ or $t_3 = (t_1+d) + d = t_1 + 2d$

$t_4 = t_3 + d = (t_1+2d) + d = t_1 + 3d$

$t_1 = t_1$

$t_2 = t_1 + d$

$t_3 = t_1 + 2d$

$t_4 = t_1 + 3d$

You can see the pattern emerging. Write down t_5 and t_6 in the same way.

What's the seventh term in the AP 3, 7, 11 ...?
We need the first term and (7-1) = 6 differences:
$t_7 = t_1 + 6d = 3 + 6(4) = 3 + 24 = 27$.

What about working backwards? If you have the term and want to know which one it is? We'll call the one we're looking for t_n (n stands for "any number"). Which term is 47 in the AP above?
For t_n we need (n-1) differences; $t_1 = 3$ and $d = 4$.
so 47 is $3 + (n-1)\,4$, or $44 = (n-1)4$, $n-1 = 11$ and $n=12$.

We call t_n the "general term" and to find it for any AP all we need is this formula (the pattern we have found)

$$\mathbf{t_n = t_1 + (n-1)d}$$

What's the 101st term in the AP 50, 49, ...
$t_1 = 50$ and $d = -1$
$t_{101} = t_1 + (101-1)(-1)$
$t_{101} = 50 + 100(-1)$
$t_{101} = 50 - 100 = -50$.

Exercise 5

Use the formula to answer the questions.
1) find t_9 for 2, 9, 16, ...
2) find t_7 for 13, 15, 17, ...
3) find t_{101} for 1, 8, 15, ...
4) find t_{201} for 1, 3, 5, ...
5) find t_{21} for 200, 197, 194, ...
6) find t_9 for 13, 9, 5, ...
7) find t_{11} for 1, 2·5, 4, 5·5, ...

(Note: most algebra text-books use the letter "a" for the first term of a progression; I think t_1 is more logical.)

So far we've given you the numbers you need to find the answer. In these three questions you have to find your starting numbers.

8) A man saves £80 in the first year and each year afterwards he saves £15 more than in the preceding year. How much does he save in the 10th year?

9) A man starting business loses £240 in his first month, £160 in the second month and £80 in the third. If this trend continues over the next few months will he be making money by his twelfth month - and how much?

10) A marble rolls down a sloping groove, travelling 3″ in the first second, 9″ in the next and 15″ in the third. If it continued like this how far would it travel in the 11th second?

AND ON WE GO ...

There are four items in our formula: t_1 (the first term), d (the common difference), n (the number of terms) and t_n (the general or "nth" term). Given any three of these we could be asked for the fourth; that's what we'll look at now.

Finding the first term.
Suppose we know that in an AP the common difference is 2 and the fourth term is 7, and we want to know the first term. List what we know:
$t_4 = 7$ and $d = 2$.
We could work backwards, if there are only a few terms:
So $t_4 = t_3 + 2$ or $7 = t_3 + 2$, then $t_3 = 5$
$t_3 = t_2 + 2$ or $5 = t_2 + 2$, then $t_2 = 3$
$t_2 = t_1 + 2$ or $3 = t_1 + 2$ and so $t_1 = 1$.

But this would be a very long-winded way of working especially if we were given, say, t_{1000} and d!
In Mathematics we look for quick ways to solve problems (and if possible, elegant ways) and for the APs we can speed things up by using the formula we found earlier.

$$t_n = t_1 + (n-1)d$$

There's our formula again.
Suppose we know that $t_{41} = 126$ and $d = 3$. What is t_1?

From the formula we know that $t_{41} = t_1 + (41-1)d$
$126 = t_1 + 40x3$
$126 = t_1 + 120$
so $t_1 = 6$.

Exercise 6

Find t_1 if:
1) $t_7 = 7$ and $d = 1$
2) $t_3 = 7$ and $d = 2$
3) $t_3 = 7$ and $d = 3$
4) $t_{21} = 41$ and $d = -½$
5) $t_6 = 12$ and $d = 2$
6) $t_7 = 20$ and $d = 3$
7) $t_6 = 29$ and $d = -4$
8) $t_{10} = 5$ and $d = -2$
9) $t_5 = 6·8$ and $d = 0·2$
10) A man saves a certain amount of money in his first year and then £100 more each year after that. If he saves £1,125 in his twelfth year how much did he save in the first year?
11) A bookworm, athirst for knowledge, munches his way through a certain number of pages in the Encyclopædia Britannica and then, getting tired, eats his way through 2 pages less each succeeding hour. In the fourth hour, worn out by his exertions, he eats through six pages. How many pages did he work his way through in the first hour?

FINDING THE COMMON DIFFERENCE.

Now suppose we are told the first term and any other term and asked to find the common difference. If the terms aren't the first and second (that would be too easy ...) then we can use the formula.
Suppose $t_1 = 7$ and $t_{11} = 150$.
Put the values we know into the formula:

$t_1 = 7$, $t_{11} = 150$, so $n = 11$ and $(n-1) = 10$:
$150 = 7 + 10d$
or
$143 = 10d$
and $d = 14{\cdot}3$.
We can now reconstruct the series:
7, 21·3, 35·6, 49·9 and so on.

Exercise 7

Find d if:
1) $t_1 = 2$ and $t_3 = 10$
2) $t_1 = 2$ and $t_8 = 23$
3) $t_1 = 3$ and $t_5 = 23$
4) $t_1 = 24$ and $t_9 = 0$
5) $t_1 = 4$ and $t_6 = 14$
6) $t_1 = 10$ and $t_{10} = 100$
7) $t_1 = 40$ and $t_5 = 16$
8) $t_1 = 7$ and $t_9 = 9$

9) A man saves £100 in his first year and £280 in his tenth year of saving; if he increases his savings each year by the same amount, how much is that amount?

FINDING N, OR THE NUMBER OF TERMS

For this we need to know the first term, the common difference and some other term.
For example take $t_1 = 1$, $d = 4$ and we want to know which term of the AP has the value 21 - if $t_n = 21$, what is n?

$t_n = t_1 + (n-1)d$

$21 = 1 + (n-1)4$

or $20 = (n-1)4$

so $5 = (n-1)$ because $4 \times 5 = 20$

and $n = 6$.

Exercise 8

What is the value of n if:

1) $t_1 = 6$, $d = 1$ and $t_n = 8$
2) $t_1 = 8$, $d = 3$ and $t_n = 23$
3) $t_1 = 9$, $d = -4$ and $t_n = -31$
4) $t_1 = 4$, $d = 2/3$ and $t_n = 18$
5) $t_1 = 2$, $d = 3$ and $t_n = 302$
6) $t_1 = 10$, $d = 6$ and $t_n = 58$
7) $t_1 = 100$, $d = -3$ and $t_n = 70$
8) $t_1 = 1$, $d= 0{\cdot}3$ and $t_n = 3{\cdot}1$
9) $t_1 = 100$, $d = -\ 1{\cdot}2$ and $t_n = 82$
10) The temperature of the water in a boiler is rising at a steady rate of 6ºF every 20 minutes. Readings taken every 20 minutes are as follows: 82º, 88º, 94º ... If the last reading taken was 190ºF how many readings were taken?
11) In a pile of timber each horizontal layer congains 3 beams more than the layer above it. If there are 70 beams on the top layer and 376 on the bottom layer, how many layers are there?
12) A swimming bath has a sloping floor and the depth of water is shown by posts placed at equal intervals down the side of the bath. The reading on the first post is 14′, on the second 13′8″. How many posts are there if the last one reads 4′?

> Ah, but my computations, people say
> Have squared the Year to human compass, eh?
> If so, by striking from the calendar,
> Unborn Tomorrow and dead Yesterday...
>
> from *The Rubaiyat*, Omar Khayyám

NEW LAMPS FOR OLD - OR MAKING A NEW AP OUT OF AN OLD ONE

The AP 1 3 5 7 9 can be made into a new one
1 5 9 by leaving out every other term.
We could also make a progression into a new one by adding terms between pairs
1 11 21
1 6 11 16 21.

How do we know what to insert?
Suppose we have the AP 1, 9, 17 ... and we're going to insert one term between each pair to make a new AP. So we want the AP 1 ... 9 ... 17 ... 25 ... with new terms where the dots (...) are. We could use the formula by taking t_1 as 1 and t_3 as 9, with t_2 to be found.
By trial and error you should be able to see that we need to put 5 between the 1 and the 9 and 13 between the 9 and the 17, giving us a difference of 4 for the new AP.
Do the same for 1 ... 11 ... 21 ... 31,
and for 6 ...12 ... 18 ... 24.

You should get d=5 for one and d = 3 for the other.
As the new term has to be halfway between the given terms then you may have realised that what you need is to work out the mean (the "average") of the terms on either side. 1 + 9 = 10, and half this is 5: 1, 5, 9, ... 1 + 11 = 12 and half this is 6: 1, 6, 11, ... and lastly 6 + 12 = 18, half this is 9: 6, 9, 12 ...

If, however, we wanted to insert two terms in this first series we could not use this idea of the mean - but there are other ways of finding the answer.

From the AP 1, 13, 25, ... we shall create another AP by inserting two terms inbetween each of those in the given set.
We start off with t_1= 1, t_2 = 13, and d = 12. Looking at the new AP:
1, ..., ..., 13,..., ..., 25
By inserting two terms between 1 and 13 then 13 is t_4 in the new AP and we have to find t_2 and t_3.
What do we know?
We know that $t_4 = t_1 + 3d$
or $13 = 1 + 3d$
so $12 = 3d$ and then $d = 4$.
So our new AP has t_1=1 and d = 4, and we now have the AP
1, 5, 9, **13**, 17, 21, **25** ... (the terms in the original AP are printed in bold type).

Here's a second example - turn the AP 16, 160 into a new one by putting 23 terms between 16 and 160.
t_1 = 16, and in the new AP 160 must be t_{25}, and $t_{25} = t_1 + 24d$
$160 = 16 + 24d$
$160 = 16 + 144$
$24d = 144$
$d = 6$
and the new AP is 16, 22, 28, 34 ... 154, 160, ...

Exercise 9

Insert one term between each pair of the set to make a new AP
1) 9, 15, 21, 27
2) 7, 17, 27, 37
3) 43, 41, 39, 37
4) 8, 9, 10, 11
5) 11, 8, 5, 2

Exercise 9 continued

insert terms between those given to make new APs:

6) 1, 7, 13 ... insert two terms
7) 1, 13, ... insert three terms
8) 2, 14 ... insert three terms
9) 100, 80, ... insert three terms
10) 55, $56\frac{2}{3}$, ... insert four terms
11) a man has a debt to pay and agrees to pay it in five instalments; the instalments form an AP. If the first payment is £50 and the last is £78, what are the other payments?

We said that inserting one term meant finding the mean of two terms. Given the set {1, 13, 25...} we find the mean of 1 and 13 to get the second term for our new AP: $(\frac{1}{2})(1+13) = (\frac{1}{2})(14) = 7$. Note what happens to the common difference, d. In the original set it was 13 - 1, or 12. In thew new set it is 7 - 1 = 6. Now 6 is half twelve, so the original d has been halved. Can we use this idea to save ourselves some work? Can we apply a similar idea to cases where we insert 2 terms, or 3 terms or more?

Starting with {1, 13, 25, ...} find what happens to d when we form a new AP by inserting:

a) 2 terms between each pair
b) 3 terms between each pair
c) 5 terms between each pair?

Now ask yourself what fraction each new d is of the old one. In each case the old (original) d was 12; what fraction of 12 is each of the new ones in (a), (b) and (c)?

Summarising you should have found

Original series:	1, 13, 25 ... with d = 12
with 1 term inserted:	1, 7, 13, 19 ... with d = 6
with 2 terms:	1, 5, 9, 13, ... with d = 4
with 3 terms:	1, 4, 7, 10, 13,... with d = 3
with 5 terms:	1, 3, 5, 7, 9, 11, 13, ... and d = 2

We didn't ask you for the result when you insert 4 terms... what is it?

What pattern have we found?

inserting 1 term	d = 6	6 = ½ of 12
inserting 2 terms	d = 4	4 = ⅓ of 12
inserting 3 terms	d = 3	3 = ¼ of 12
inserting 5 terms	d = 2	2 = ⅙ of 12.

And to complete the pattern, when we insert 4 terms, we find d = 2·4, which is (check it) one-fifth of 12.

If you look at the denominators of the fractions you should see a pattern:

number of terms inserted	denominator of fraction
1	2
2	3
3	4
4	5
5	6

which suggests for the general number of terms, n

n	n+1.

Check: in the AP 1, 17 we are going to insert 7 terms. The rule we have discovered tells us that the difference for the new series will be $1/(7+1)$ or ⅛ of the original difference.

Is this true? check it for yourself.

Exercise 10

1) insert terms between 8 and 20 to make an AP of 5 terms
2) insert terms between 16 and 100 to make an AP of 15 terms
3) insert terms between 500 and 20 to make an AP of 25 terms

We are nearly at the end of the work on the AP and its terms. We have laid most stress on examples in numbers but, of course, you should be able to solve problems expressed in words as well. Exercise 11 uses what you have learnt in a miscellany of problems which will require some thought to answer.

Exercise 11

1) How many numbers are there between 1 and 1,000 thich can be divided exactly by 3?
2) How many three-figure numbers are there which are divisible by 7?
3) How many whole numbers are there between 1 and 1,000 which leave a remainder of 3 after being divided by 4?
4) A man starts with a salary of £12,000 a year and receives annual increases of £150. How much will his salary be during his twelfth year of service?
5) A man saves £75 one year, £120 the next and £165 in the third year. How much will he save in his tenth year if he continues to save in this way?
6) A clerk addresses envelopes at the rate of 120 and hour for the first hour but, as he grows tired, his speed decreases by 12 envelopes an hour . How many envelopes does he address in the 5th hour?
7) In “free fall” a body drops 16 feet in its first second of falling. In each second following it falls 32 feet more than in the previos second. How far will it fall in the 7th second? and in the 14th?
8) The temperature inside the Earth increases by 2°F for every 300 feet nearer the centre of the Earth. At a depth of 75 feet the temperature is 50°F. What is the temperature at a depth of 2,775 feet? At what depth is the temperature 450°F (when tin melts)?

Exercise 11 continued

(NB we are treating temperature rise here as if it were a "step graph"; in reality the temperature change would be continuous).

9) When an object is thrown straight up it travels 16 feet less in each second than in the previous one. If it reaches a height of 240 feet at the end of the first second how high will it go in the second? how high in the third? And in which second will it stop travelling upwards?

Here are some progressions which are NOT in base ten. In what base is each written?

10) 3, 6, 9, 10, 13, 16

11) 2, 4, 6, 8, 11, 13, 15

12) 4, 10, 14, 20

13) 3, 6, 12, 15, 21, 24

14) 4, 8, 10, 14, 18, 20

15)1, 2, 3, 4, 10, 11, 12

16) 0·3, 0·6, 0·9, 1·0

17) 1, 10, 11, 100, 101

Orderings

In the series 1, 3, 5, 7 the terms increase by 2 each time; from what point on are the terms larger than 100?

$t_1 = 1$ and $d = 2$ and what we are asking for is the value of some term tn where $t_n > 100$.

$t_1 + (n-1)d > 100$

$1 + (n-1)2 > 100$

$(n-1)2 > 99$

$n-1 > 49\frac{1}{2}$

$n > 50\frac{1}{2}$

as 51 is the next whole number after 50½ we know that this means that t_{50} is less than 100 and t_{51} is more than 100.

Check

$t_{50} = 1 + (49)2 = 1 + 98 = 99$ and $99 < 100$

$t_{51} = 1 + (50)2 = 1 +100 = 101$ and $101 > 100$.

Exercise 12

from what point on are the terms of the AP given greater than the number given?

1) 1, 7, 13 ... greater than 100
2) 5, 13, 21 ... greater than 200
3) 15, 50, 85, ... greater than 1,000

and from what point on are the terms of

4) 100, 88, 76 ... less than 10?

Inserting terms, an extra note.

We said that the method of averaging two terms would not work for inserting two terms. It can, however, be adapted for three terms.

Suppose we want to insert three terms between the numbers 1 and 13:

1, ..., **X** , ..., 13, ...

this term is the mean of 1 and 13, i.e. 7

1, **X**,. 7, ..., 13, ...

this term is the mean of 1 and 7, i.e. 4

1, 4, 7, **X**, 13, ...

this term is the mean of 7 and 13, i.e. 10

1, 4, 7, 10, 13.

You might like to investigate this further. It works for inserting one term and for inserting three terms. For what other number of inserted terms will it work? Is there a set to which they belong?

Use this method to insert seven terms between the numbers 1 and 33.

You have now completed the first workbook in this series. The second book continues the work we have started and goes on to discuss the sums of the terms of an AP and what they in turn lead on to.