# DOZENS ARITHMETIC FOR EVERYMAN

BY

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WITH ADDITION AND MULTIPLICATION TABLES

THE DOZENAL SOCIETY OF GREAT BRITAIN

## DOZENS ARITHMETIC FOR EVERYMAN

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If you ever had the job of packing the hired cups after a charity fête, and counting the empty soft-drink bottles, or if you have assisted a grocer or wholesaler with stocktaking, it is quite probable that you tallied things by the dozen. This would have been natural since they were probably packed by the dozen. Things usually are, because experience has taught us that this is the most efficient and economical way. There are many reasons which we shall not go into fully here, but briefly, packing by the dozen (or numbers simply related to it) gives a better combination of strength, ease of handling, flexibility and economy of materials than any other way. For instance, twelve equal-sided blocks can be packed in a box which requires less material than one for ten.

The nett result is a practical advantage and a saving in cost in packing by dozens. In a few cases - cigarettes, which are thin, or razor blades, which are flat - this advantage is slight, but these are the exceptions, not the rule.

### Counting by Dozens

If you counted things by dozens as they went neatly into their crates or packs, and the smaller packs went into larger ones (packets of matchboxes being an example) you may have found yourself counting by dozens of dozens. This is just as easy and of course we have a name for a 'dozen dozen', namely a 'gross', so that, having counted one gross you could start again with the dozens of the next gross. There is no reason, either practical or theoretical, why you should not continue in this way, if there were enough things to count, reaching a dozen gross or a 'great gross'. This may be an everyday task for the grocer, whose name means "the man who deals in goods by the gross," and an American company [1] now provides mechanical dozen counters for just such purposes.

Indeed, there is a clear parallel between counting by dozens and gross and the sort children learn (probably starting on their fingers) using tens and hundreds and so on.

Let us look further at that parallel.

### Numerical Notation

When we count things we usually need to write our results down, lest they be forgotten, and so we need some kind of numerical notation. Many interesting ways of writing numbers have been invented through the ages, and much can be learnt from their study [2], but the one most widely used today is that which is directly applicable if we have been counting by tens. That is, if we have counted six hundred and seventy-four we write 674.

If we have been counting by dozens and gross because it is quicker and easier, it is a nuisance if we have to convert our result to tens and hundreds before we write it down, especially if we really want to know how many dozens there are and not how many tens.

In fact it is perfectly simple to record the dozens and gross directly, and on the stock sheets and invoices of wholesalers and wine merchants quantities are often recorded in two or three columns labelled 'gross', 'dozens', 'units'. Using them, we can record directly the number three gross, four dozens and eight as

| *gross* | *dozens* | *units* |
|---|---|---|
| 3 | 4 | 8 |

Consider, however, what might happen if a stock-clerk who was in the habit of recording quantities on stock sheets in this way temporarily ran out of the appropriate sheets of paper? It is quite likely that he would write his numbers down on blank sheets of paper, thus 3 4 8 would mean to him exactly three gross four dozen and eight.

Having done this, we may ask, how could this number be distinguished from the very different number three hundred and forty-eight, without any distinguishing mark? The answer is that it is the context or conditions in which it is used, which distinguishes the cases. (The use of context to add to meaning is very common; indeed ordinary speech would be much less intelligible without it. We all know that out-of-context quotations can appear to alter meaning).

**Positional Notation - Place Value**

When we look at any number, such as 742, in no particular context, we should probably interpret it as 'seven hundred and forty-two'. If we took exactly the same symbols and re-arranged them to get, say, 427, we should call that 'four hundred and twenty-seven' - a very different number. Clearly the context of the individual symbols, namely their relationship or place with respect to each other has made a big difference to the interpretation we place upon them.

Sometimes people take the context so much for granted that they have difficulty appreciating that they do need more information than that conveyed by the symbols alone. Nowadays, however, particularly owing to the growing importance of computers, it is becoming important to be aware of the context of numbers and it is being given much more attention in school mathematics than formerly. It is also worth noting that people can take account of changes of context. If, for instance, the timetable says that our train departs at 1102 and the time is now 1059, we know we have just three minutes to catch it, not forty-three.

If we are contrasting 742 and 427, then the way in which we understand them to differ is that the places in which the various numerals appear are different. We say that the numerals have *place value*. If we are thinking in tens and hundreds then we interpret the 7 in the first number as seven hundred and the seven in the second as seven units, because their places in the numbers are different. The place values of the numerals from right to left are units, tens and hundreds, and to emphasize this point we could rule up columns and head them appropriately thus:

| *hundreds* | *tens* | *units* |
|---|---|---|
| 7 | 4 | 2 |
| 4 | 2 | 7 |

Notice that we have used exactly the same tableau as we did earlier for recording dozens and gross, the only difference being that now our columns are headed units, tens, and tens of tens, instead of units, dozens, and dozens of dozens.

In the present case, we say that the place value is a *power of ten* and that ten is the *base, radix or scale* of notation, or that the quantity is to be understood as a decimal number.

We can treat the earlier case in precisely the same way: the place value is a *power of twelve* (or a dozen), twelve is the base or radix of notation and the quantity is to be understood as a *dozenal* or duodecimal number.

In following this argument, one may note that it is not the terms we have introduced here which are as important as the idea behind them, so if at a first reading you do not grasp what a power of a number is, that is a point which can be left meanwhile. [[3]]

The important idea is that we can use different numbers as the base on different occasions; positional notation (using place value) is no special property of ten. Any positive whole number may be used. We mentioned computers. There are very good reasons for choosing a base of two for a computer. Then we say we are using binary numbers. We can rule up exactly the same sort of tableau.

| *eights* | *fours* | *twos* | *units* |
|---|---|---|---|
| | | 1 | 0 |
| 1 | 0 | 1 | 1 |

What are the two binary numbers we have just written? Answers [4].

Other number bases often used with computers are eight and sixteen (which are simply related to two but more compact to use). Then we say our numbers are octal or hexadecimal respectively.

**Number Symbols**

If we look at the examples we have just given, we see that in binary numbers, 10 is *two* and not *ten*. Question: can you express ten as a binary number? Answer [5]. This is entirely reasonable, since in the context of the place value we have chosen, 10 is one times two and zero units, that is exactly two. Likewise three is 11 (one times two plus one unit), four is 100 and so on. One may ask: what has happened to all the other number symbols (2, 3, 4 .... )? The answer is that in binary numbers we don't need them - two numerals, 0 and 1, are *entirely* sufficient. (That is one good reason why they are attractive for computers).

Let us return now to consider numbers using the base of a dozen. In this case 10 means twelve - that is one times a dozen and no units. (In general 10 means the number being used as the base, whatever base we are using). Likewise 11 is a dozen and one, not eleven. (Do you know why 'thirteen' is often considered 'unlucky'? It is probably because this was the number who sat down at the Last Supper. In dozenal numbers the reason is quite apparent).

In this case, you may say, what are we going to do to represent ten and eleven? The answer is that two added symbols must be provided - base twelve needs twelve distinct symbols, including the zero. There is no real difficulty in doing this. For example, in a sterling adding machine the shilling and pence columns are exactly the dozens and units columns of our first tableau. [6] In such machines ten and eleven pence are generally indicated by special symbols which are 10 and 11 where the digits are packed up very close to each other, effectively forming two new symbols distinguishable from any other use of 1 and 0 independently.

For general purposes, however, we really need two much more distinct symbols. Any two fulfilling this condition would do. This has been a rather fertile field of invention and in practice rather a wide range of symbols has been used. Of course it will settle down with usage. (It has done so for the other ten numerals, although considerable variation continues to exist). Some authors use 'T' and 'E' or 't' and 'e', the initial letters of ten and eleven; others have used 'X' and 'Y' (Roman ten is X).

With the IBM360 series of computers, hexadecimal (base sixteen) numbers are often used. This requires a total of sixteen symbols and ABCDE and F have been pressed into service for the numerals ten to fifteen. The advantage of using letters is that they are readily available on typewriters and elsewhere. The disadvantages are that they don't 'look like' number symbols and there is always a risk of confusion like that we have already sometimes between zero and the letter 'O'.

Here we shall use the symbols ↊ (an inverted 2) and ↋ (an inverted 3). Confusion is still possible like that occasionally arising between 6 and 9 but we have two distinct symbols. They were originally chosen by Sir Isaac Pitman, a keen advocate of dozenal arithmetic. [7] They are quite easy to remember because ↊ has a certain resemblance to a script T and ↋ to a script E, for 'ten' and 'eleven', respectively.

### Number Words

Number words, like any other words, are the product of the continuous process of the evolution of language. This is a phenomenon which the proponents of any artificial system of nomenclature would do well to recognize (an example is the nomenclature of the metric system which has changed considerably since first specified. It is now less systematic than formerly and it still lacks much of the succinctness of words in the foot-pound system).

Natural usage has tended to give us more words for decimal numbers than any others (e.g. hundred, million) because decimals have most generally been used for *talking about* numbers, even though the majority of *uses* of number have not been decimal and indeed (with computers especially) are not so today either. Of course there are other number words such as we have used already (dozen, gross, great gross, score), but more words for non-decimal bases would be useful.

Original thinkers, interested in developing the practical potential of the dozen as a number base, have been attracted to this field. As a result, a number of interesting proposals has arisen, most of them in fact quite simple to use, but also perhaps, because of the new words they contain, slightly alarming in their unfamiliarity at first sight.

Because they are not essential to the ideas of dozens arithmetic and are not essential to our present discussion we shall not use them here. Fortunately, because of our many years' use of shillings and pence we have a naturally evolved practice which will do very well for many cases. We can also, if we wish, use the practice of calling out the individual digits, as we quite often do when stating a telephone number or car or house number of three or more digits.[8] For example, 6946 becomes simply 'six nine four six'.

Here are some suggestions.

26 'two dozen & six' *or* 'two & six'(everyone knows there are two dozen & six pence in half a crown)

60 'six dozen'

457 'four gross, five dozen & seven' *or* 'four gross, five & seven' *or* 'four five seven'

1↊ 'a dozen & ten' *or* 'one & ten'. If we take two pence from two shillings this is the number of pence remaining, of course, using our special symbol for 'ten'

8↋9 'eight gross, eleven dozen & nine' *or* 'eight eleven nine'

1008 'a great gross & eight' *or* 'one nought nought eight'

Try some, answers [9]:

(i) 10 (ii) 13 (iii) 123 (iv) 209 (v) 89↊ (vi) ↋↋↋

It is fair to say that only a very slight extension of existing practices is required in order to talk about dozenal numbers with ease and facility.

### Distinguishing the Base

Sometimes when it is not obvious from the context, or when we wish to distinguish between two bases at the same time, special means are adopted for the purpose.For example, the base may be written as a Roman number in small type after a number. Thus $776_{VIII}$ is the octal (base eight) number 'two less than $1000_{VIII}$'. (Why do we know that 789 couldn't possibly be octal?)

Sometimes dozenal numbers are distinguished by being preceded by an asterisk, and decimals by a dagger, or different type styles are used.

### Dozenal Addition

For arithmetic using any base it is necessary to know the addition table in that base. It is probably true to say that shop assistants and others using shillings and pence use their dozenal addition tables almost to the exclusion of any others, and that a population at large familiar with shillings and pence is well aware of its dozenal addition tables. It is a serious criticism of the general system of elementary education that very seldom is it ever pointed out that the addition rules learnt for shillings and pence are not special cases at all, but are perfectly generally applicable to all cases of reckoning in dozens. As these include many measurements of time and length as well as the actual tallying of dozen packs we described earlier, they are often more widespread in their usefulness than decimal addition tables.

Thus, people in Britain know their dozenal addition tables already and all we need do here is give further examples of their use in everyday situations.

| | | | | | |
|---|---|---|---|---|---|
| | 7 | 19 | 64 | 85 | 76 |
| | 8 | 23 | 26 | 76 | 56 |
| sum: | 13 | 40 | 8Ɛ | 13Ɛ | 110 |

If we think of these examples as columns of shillings and pence then each (with the possible exception of the last two) is entirely familiar.

7d + 8d = 1s 3d.  1s 9d + 2s 3d = 4s 0d.

6s 4d + 2s 6d = 8s Ɛd (remember we have used our distinct symbol for ten but said nothing new, only ‘six and four plus two and six equals eight and ten’).

Here the s. and d. are not necessary to the arithmetic; we have just put them in because we were talking about money.

The third column from the right in our last example may look a little less familiar, because a column containing gross is not normally used for money. The trick is first to think of the right-hand column as pence (6d + 6d is 1s 0d, write down 0 and carry 1), *then* to think of the next column as pence (7d + 5d + 1d (carried) equals is 1s 1d, write down 11). One can extend this across any number of columns, because the addition rule for each column in dozens arithmetic is exactly the same as if it were pence, and this property can be applied consistently, that is, one does not have to change one’s thinking from dozens to scores to tens.

The same arithmetic as in these examples is applicable in many other places. For instance, in addition of lengths:

7in + 8in = 1ft 3in.  1ft 9in + 2ft 3in = 4ft 0in, and so on.

Likewise, 7 months + 8 months = 1 year, 3 months.

1 year 9 months + 2 years 3 months = 4 years 0 months.

Or 8 months after July (the 7th month) is March (the 3rd month) of the next year (indicated by the 1 carried). How often, instead, do people do this on their fingers!

8 hours after 7 a.m. is 3 p.m. This example indicates a much more attractive way to express times, without using ‘a.m.’ and ‘p.m.’, than the ‘twenty-four-hour clock’ or ‘military time’ adopted by British Rail and some other organizations, against the general wishes of the people at large. This is clumsy because it uses decimal notation to talk about things measured in dozens and is one of many instances of the handicap to using our efficient and practically evolved units of measure with the inappropriate decimal notation.

Thus since there are two dozen hours in a day, each of the morning hours can be expressed by one of our twelve dozenal numerals (0 to Ɛ), preceded if necessary by a zero, and each of the afternoon hours by one of the same set of a dozen numerals preceded by a one. Thus 8 o’clock is simply 800, 8 a.m. is 0800, and 8 p.m. is 1800. Clearly the asterisk or something similar would sometimes be needed to distinguish this from military time (perhaps inserted between the hours and the minutes; e.g. 18*00 = 8 p.m. or 2000 h. in decimals). Of course, there is nothing inherently wrong with continuing to use ‘8 a.m.’ and ‘8 p.m.’ and we can still use dozens arithmetic conveniently in conjunction with them.

**Dozens subtraction** is as straightforward as addition so we simply give one example:

13 - 8 = 7.

Whence, for example, eightpence less than 1/3 is sevenpence.

Examples, answers [↊]. Remember to think of each column as pence]:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (i) | 9+ | (ii) | 28+ | (iii) | 149+ | (iv) | 789+ | (v) | 63- |
| | 6 | | 66 | | 283 | | 222 | | 26 |

**Dozenal Multiplication**

Recalling the decimal multiplication tables - only the two, five, ten and eleven times were easy to learn; no simple patterns appeared to assist with the others. As a result of this difficulty many people fail to master these tables properly and others who could correctly quote the decimal product of, say, 6 x 4 if asked, have insufficient mastery to be able to apply their multiplication in practical cases, such as finding the number of stamps in a block six by four. As a result there is much unnecessary use of ready reckoners or desk calculators which is often slower than mental arithmetic and sometimes more prone to mistakes. There may even be a reversion to counting items individually.

What happens when dozens are used? Let us consider our original packing or stock-taking problem. We might count our dozens by ones or twos and this would hardly differ from decimal practice, but equally if things are packed in rows of three, we might find ourselves saying: 'three, six, nine, a dozen, (one and) three, (one and) six, (one and) nine, two dozen' and so on, where we might or might not trouble to include the 'one and's bracketed above.

Alternatively, if we were tallying rows of four, we might proceed: 'four, eight, a dozen, (one and) four, (one and) eight, two dozen, etc. or even, if we could conveniently see groups of six at a glance (and this is quite common, say in double rows of three), we could say: 'six, a dozen, one and six, two dozen, two and six, three dozen, .. etc.'

Because each of the steps we have chosen divides exactly into a dozen we get simple repetitive patterns which are easy both to learn and apply. By contrast, counting tens by threes, fours or sixes is complicated to the extent that it is hardly ever done in practice, and counting by fives, which would he easier, is likewise uncommon because rows of five are less common or harder to spot at a glance. Thus decimal counting is seldom done by more than twos and is correspondingly slow on that account.

Of course counting by dozens is not restricted to tallying things packed in dozens. Many chemists, for example, have discovered it is faster for counting pills into bottles. You could try it for counting the words on this page. It is quite easy to count words in groups of three (by dozens), but on account of their variable sizes most people find it difficult to recognize larger groups of words quickly.[↋] Unless you are unusually gifted, therefore, you will probably find you can count these words faster by dozens in groups of three than any other way. Afterwards try the same with any one kind of coin.

So far we have not seemed to say very much about dozens multiplication, but in fact we have already presented several of the tables. For example in counting by fours, 'four, eight, a dozen, one and four, etc.', we have just been stating entries in the four times table in dozens, for these numbers are precisely four 1s, four 2s, four 3s, four 4s, and so on. The 2, 3, 4 and 6 times tables are just those from which we have quoted. A similar simple pattern is evident for 8s, 9s, and twelves. For ↋s the pair of figures in the result always adds up to eleven, e.g. three elevens are two and nine. This leaves the 7 times table, which is no more simply related to twelves than to tens, but less commonly used than other tables, and, as is to be expected, 5s and ↊s which are harder than in decimals. Even so, they are not entirely unfamiliar. Just as 5 fivepences are 2s.1d., so 5 times 5 is 21 in dozens.

In fact the dozenal multiplication tables are often present in disguise in ready reckoners for shillings and pence; most people are familiar with all the entries in them and can instantly recall many of them (six fours are two dozen for instance). Learning them undisguised of course, one ceases to need the ready reckoner and is able to apply them to many other things besides money calculations.

The possibilities of simplifying calculations using dozenal multiplication are very great, and we give a small number of examples here.

| | | |
|---|---|---|
| | 14 | 3 x 4 = 10, so write down 0, carry I |
| x | 23 | 3 x 1 = 3, so add carried 1, write 4 |
| | 40 | 2 x 4 = 8 |
| | 28 | 2 x 1 = 2 |
| | 300 | Then add the columns. |

There are numerous immediate applications of this example;

e.g.: 1ft 4in @ 2s 3d per ft costs 3s 0d

2ft 3in @ 1s 4d per ft costs 3s 0d

1 dozen & 4 items @ 2s 3d per dozen cost 3s 0d

1ft 4in by 2ft 3in is 3 square feet or 300 (3 gross) square inches.

Try these examples, [[10]] and interpret them similarly:

(i) 28 x 36

(ii) 19 x 34

There is one particular (and easy) case of dozenal multiplication which is not only important in itself but also for the general principle which it illustrates. Once again it is familiar to us in the form of a special case - the well known 'dozen rule'. As an example, the price of a dozen articles at 7d each is 7s 0d.

Generally, 7 x 10 = 70, and again, 863 x 10 = 8630.

Applying the rule twice, 94 x 100 = 9400; 2Ɛ x 100 = 2Ɛ00.

All of this looks absurdly obvious in the context of decimals, e.g. to multiply by a hundred, add two zeros. What is not widely recognized is that there is an exactly equivalent rule for dozens, i.e.to multiply by a gross, add two zeros.

This 'place-shifting' property of multiplication (and division) by the base is indeed quite general for any number base (try it for three times two in binary numbers). It is certainly not a special property (there is none) of the number ten.

We shall find further applications of dozens multiplication after considering fractions.

**Dozenal Division**

Direct division in dozens is quite straightforward. We shall not develop it here, but illustrate some cases in which division may simply be replaced by multiplication.

For example, since two is a dozen divided by six, division by six is the same as multiplication by two and then division by a dozen. Since division by a dozen (the inverse of multiplication) is done by simply crossing off the last zero (when present), the work in the division is replaced by an easy multiplication. (If the last figure is not zero, the result of the division will include a fraction but this is always possible, whatever base is used).

Examples: 136 ÷ 6 = (136 x 2) ÷ 10 = 270 ÷ 10 = 27.

83 ÷ 6 = (83 x 2) ÷ 10 = 146 ÷ 10 = $14\frac{6}{10}$ which is $14\frac{1}{2}$

since $\frac{6}{10}$ is six twelfths or one-half.

There are numerous cases where multiplication and division are simply interchangeable. Many of these are for divisions often needed, and in most cases the alternative multiplication is simpler in dozens.

Division by 2 is equivalent to multiplication by 6 & 'crossing off' 1 figure
Division by 3 is equivalent to multiplication by 4 & 'crossing off' 1 figure
Division by 4 is equivalent to multiplication by 3 & 'crossing off' 1 figure
Division by 6 is equivalent to multiplication by 2 & 'crossing off' 1 figure
Division by 8 is equivalent to multiplication by 16 & 'crossing off' 2 figures
Division by 9 is equivalent to multiplication by 14 & 'crossing off' 2 figures
Division by 14 is equivalent to multiplication by 9 & 'crossing off' 2 figures
Division by 16 is equivalent to multiplication by 8 & 'crossing off' 2 figures

The 'crossed off' figures are the fractional part. If this is to be retained we need to know something of fractions.

**Dozenal Fractions**

As for any base, fractions may be expressed in dozens either as vulgar fractions or by using the place-value method.

For vulgar fractions all we need to remember is that both top and bottom lines are dozenal numbers. Thus three-eighths is simply 3/8 and half of that is 3/14 since twice 8 is 14.

Using the place-value method, the leftmost figure in a fractional part represents the twelfths and the next to its right the parts per gross. A separator or 'point' is needed to distinguish the fractional from the whole number part. Dozenal practice here shows perhaps less variation than in decimals, for which . · and , are variously used and disputed. In dozens : or ; is commonly used and we shall use the latter here. (The shilling symbol / could be used, since pence are twelfths of shillings).

One drawback in the use of the place-value method for fractions is that, whatever base is used, recurring numbers can arise. An important advantage of using dozens is that, in practice, these are notably less frequent than the 'damned dots' which occur so often with decimals.

The simplest fraction, one half, being six twelfths, is written 0;6. This is entirely in accordance with experience - sixpence is half a shilling, six inches half a foot, six months half a year, and at 'half past' the larger hand on the clock face points to six.

Half of a half is a quarter and half of 0;6 is 0;3 and so a quarter is 0;3 and we have an exactly similar set of examples to those for six above. Similarly three-quarters - a half plus a quarter or three times 0;3 is 0;9, so each of the quarters is a one-figure number.

Considering next one-third, since this is exactly four twelfths, we write it simply 0;4. Likewise two-thirds is 0;8. A third of a year is four months, two-thirds of a shilling is 8d.

The fifths are recurring fractions - one fifth being very nearly 0;25. The sixths however are very simple, one-sixth being 0;2 and five-sixths 0;↊ (one sixth of a foot is two inches, but we shall not labour the point).

Sevenths are awkward but not used much. Eighths are important and need attention. Now a half of 3d is 1½d or 1;6d or 0;16s. Alternatively half of 3ft 0in is 1ft 6in. In short, this is our table of eighths:

$\frac{1}{8}$ = 0;16 $\frac{3}{8}$ = 0;46 $\frac{5}{8}$ = 0;76 $\frac{7}{8}$ = 0;↊6

each differing from its neighbour by 0;30, that is, one-quarter.

Halving eighths again, sixteenths (or dozen and fourth parts) show a significant saving over decimals. Thus, one-sixteenth 0·0625 decimal = 0;09 dozenal, three-sixteenths = 0·1875 decimal = 0;23 dozenal.

This trend continues for all the fractions created by successive halving - so important for computer calculations and in many other spheres. The dozenal fractions need about half as many figures as the decimal ones. Tenths and twelfths exchange rôles compared with decimals. Tenths recur but one twelfth is simply 0;1.

Comparable to the derivation of percentages for decimal fractions, we obtain 'pergrossage' in dozens. We exchange 12½ per cent for 16p.g. (one and six per gross), and 33⅓ p.c. for 40 p.g. (four dozen/gross).

**Operations with dozenal fractions**

We are now able to widen our rules for multiplication and division by a dozen to include fractions. To multiply by a dozen, shift the dozenal point one place right; to divide by a dozen shift it one place left. The extension to multiplication and division by the gross and so on is obvious. Thus,

12;46 x 10 = 124;6          2Ɛ9 ÷ 100 = 2;Ɛ9

In mu1tiplying numbers containing dozenal fractions the only extra thing we need apply is a rule we know already to keep track of the point. Add the number of digits to the right of the point in both factors to find the number to the right in the product.

Example 1. (a) find the cost of hire for 2 hrs, 20 minutes @ 3/9 per hr.

(b) find the area of a piece of material 2′4″ by 3′9″.

The working of these is identical, since 2 hours 20 minutes is 2;4 hours (at 20 minutes past the hour, the minute hand is at 4). We have:

```
     3;9
x    2;4        Answers:   (a)   8/9d
    ----
    130
     76                    (b)   8¾ sq. ft.
    ----
    8;90
```

Example 2. (a) find the cost of a sheet of glass 1′2″ by 1′6″ @ 3/8 per square foot.

(b) find the volume of a box 1′2″ by 1′6″ by 3′8″.

Once again the working is identical:

```
     1;2
x    1;6
    ----
      70
     12         Answers:   (a) 6/5d
    ----
     190
x    3;8                   (b) 6;5 cu. ft.
    ----
    1200
    530
    ----
   6;500
```

Example 3. Changing from cubic feet to superfeet is merely a shift of the dozenal point. (A superfoot, or 'superficial' foot, commonly used in timber measurement, is 1′ x 1′ x 1″). All the timber merchants' 'rules of thumb' are normal rules of arithmetic in dozens. Find the volume in (a) cu.ft., (b) su.ft. of a piece of timber 8′6″ by 8″ by 2″.

```
0;8 x 0;2 = 0;14        8;6
                     x 0;14
                     ------
                        2↊0      Answers:
                        86       (a) 0;Ɛ4 cu.ft.
                     ------
                      0;Ɛ40      (b) Ɛ;4 su. ft. i.e. eleven & one third su. ft.
```

Try these exercises:

1. (a) the cost of $4\frac{1}{2}$ dozen articles at 1/2d per dozen;

   (b) the area of a surface 4′6″ by 1′2″.

2 (a) the cost of a sheet of materaI 1′4″ by 1′3″ @ 4/3 a sq.ft.;

   (b) the volume of a crate 1′4″  by 1′3″ by 4′3″.

3. the weight of a sheet of metal 1′5″ by 1′9″ at 6oz per sq.ft.

4. the freight on a carton 1′4″ by 1′6″ by 3′6″ at 6/2 per cu.ft.

Notes: Minor conversions are sometimes convenient after working in dozens; these are illustrated in our answers to 3 and 4 (below).

We should be surprised if everybody attempting these exercises got them right first time - think of all the time you spent learning decimal arithmetic! However when you *do* get the right answers you may fairly consider you have got the idea of dozens arithmetic correctly.

Answers: 1.(a) 5/3d. (b) $5\frac{1}{4}$ sq.ft. 2.(a) 7/1d. (b) 7;l cu.ft.

3. 12;↊6 oz. ($14\frac{7}{8}$ oz. decimally) 4. 37;2 shillings (£2 3s 2d).

**Conclusion**

The prime purpose of this booklet is educational: it is intended to be a readable survey of dozens arithmetic for the lay reader seeking an answer to the question, "Just what *is* dozens arithmetic?"

Nevertheless it would be unrealistic not to acknowledge that its effectiveness for this purpose is bound to be influenced (as any such exposition must be) by the climate - intellectual, political, economic of the times in which it is read.

Notwithstanding the rapidly increasing need for knowledge of nondecimal arithmetic and evidence of its value (largely brought about by the computer), we are currently being subjected to a heavy flood of propaganda for decimal systems. Much of this propaganda is fallacious and the thinking behind it is perhaps six generations out of date. It is simply not true that 'decimals are simpler', but the well worn principle that 'if a thing is said often enough people will believe it' seems to be propagating this belief. People are often reluctant to discard beliefs when they have eventually been accepted in this way. To counter this trend, and the barrier to enlightenment it produces, is the purpose of this summary.

1. In introducing dozens arithmetic we have based our whole development on ideas and practices which are common and widely used today. With one minor exception we have introduced no facts or concepts new to the man-in-the-street. What we have endeavoured to show is that there is a coherent pattern for many of the practices which commonsense and experience have developed, and we have tried to fit the pieces of this pattern together. We believe this to be a much better basis for everyday arithmetic than the rather thin rationality of the decimal system. The one minor innovation we have made is to introduce two new symbols for ten and eleven. It is possible that better symbols could be chosen. Written symbols are our servants, not our masters, and we can and do change them or introduce new ones when there is a need to do so.

2. Acts of parliament may make it illegal  to use the shilling and the foot, and penalties may be imposed on those who use them. (At one stage during the imposition of the metric system upon France, people were imprisoned for trading in dozens). They cannot abolish the force of practical necessity which leads people to take advantage of non-decimal methods.

3. Dozens arithmetic will always he inherently simpler than decimal arithmetic. Its margin of superiority has been estimated by Professor A.C. Aitken [[11]], until recently in the Chair of Mathematics at Edinburgh University, and one of the outstanding arithmeticians of all time. He concludes that for *everyday* kinds of calculation decimal arithmetic requires one and a half times the effort of using dozens. When one adds to this that the greater simplicity of dozens may permit a greater num-

ber of people to understand and use arithmetic effectively, the benefits of its use may be seen to be substantial.

4. An erroneous view is sometimes held that any suggestion for the use of dozens arithmetic implies an advocacy of a wholesale change of units of currency, weights and measure, and a radical and compulsory change in people's ways of thinking. This is quite untrue. We have made no sugestions for such changes here. In fact it is the decimalists (notably advocates of the metric system) who seek such a wholesale and compulsory change.

5. It is sometimes suggested that the cost and effort required to utilize dozenal arithmetic is too great. This, too, is erroneous. A complete and compulsory change need not be contemplated. It is not necessary. Consider how hexadecimal notation was adopted (with *six* new symbols) for the major IBM computer series - the System/360. This was not a matter of legislation, compulsion or massive cost, simply one of practical convenience.[12] Likewise, dozenal arithmetic is already in use - in building and quantity surveying [13], and in every shop using shillings and pence. Because it was not taught formally at school, it is often not recognized as a suitable and indeed superior form of arithmetic for most everyday arithmetic. But there is certainly no need for complete retraining of every person in the country, nor indeed for such absurdities as altering all street numbering or motor registration plates (as the secretary of the New Zealand Decimal Currency Board suggested!).

6. There is a need for development of understanding of dozenal arithmetic through the normal processes of education. Nowadays children *are* introduced to non-decimal bases at the age of nine, but they are frequently denied the opportunity to see the merit in the dozenal base. For example, base seven, of which the practical use is trivial, has been used for illustrations, and often avoidance of the dozen has been a matter of conscious policy, though the interest it arouses in children has been admitted (by Brumfield and others). This irrational opposition to its use is a real testimonial to its value.

Decimal arithmetic has always tended to be a rather 'academic' subject, divorced from much of the practical use of number, where nondecimal usage has real advantages, and the skill at arithmetic of many people has suffered in consequence. Dozenal arithmetic offers better notation, simpler methods and opportunities for higher attainment. It is a continuing challenge to the academic world to make the most of all of these opportunities.

Radlett,
Hertfordshire

**Notes:**

[ 1] Automatic Electric Company

[ 2] Peterson & Hashisaki, Theory of Arithmetic, John Wiley, 1963

[ 3] A hundred (= ten times ten) is the second power of ten;
a thousand (= ten times ten times ten) is te third power of ten;
a gross (= a dozen times a dozen) is the second power of twelve
eight (= two times two times two) is the third power of two

[ 4] Two, eleven

[ 5] 1010

[ 6]The pounds column introduces a different place value, based on the score, i.e. twenty. When this occurs we say we have a 'mixed-base system'. Nowadays mixed-base systems seem rather unpopular although they have certain advantages. Another example of a mixed-base system is that used for time, mentioned earlier. We shall indicate later how the simplest way to develop a single-base system for time, if we wanted it, would be to choose the dozen as the base.

[ 7] Pitman: The Phonetic Journal, Vol 16, 1857, page *4Ɛ

[ 8] Note that telephone numbers and house numbers are not really numbers at all, merely labels using numerals. A telephone number could be stated in letters entirely (e.g. WHI 2345, 944 2345 and WHI ADIL would all make the same connection). Likewise street numbers are sometimes mixed with letters and while they imply an ordering they don't imply any arithmetic and hence no number base. Since therefore hundreds and thousands are *not* implied in such labels we usually drop them in stating the labels.

[ 9] (i) 'a dozen' *or* 'twelve'
(ii) 'a dozen & three' *or* 'one & three'
(iii) 'a gross, two doz & three' *or* 'one two three'
(iv) 'two gross & nine' *or* 'two nought nine'
(y) 'eight gross, nine dozen & ten' *or* 'eight nine ten'
(vi) 'eleven gross, eleven & eleven'-(one under a great gross).

[ Ɀ] (i) 13 (ii) 92 (iii) 410 (iv) 92Ɛ (v) 39.

[ Ɛ] Memory 'performers' are said commonly to use patterns of three.

[10] (i) 940, (ii) 5Ɀ0.

[11] A.C.Aitken, The Case Against Decimalisation, Oliver & Boyd, 1962

[12] F.J. Budden,An Introduction to Number Scales and Computers, Longmans, 1965,

[13]. L.A. Poulden, The Duodecimal Book, Cleaver-Hume Press, 1959.

## DOZENS ADDITION TABLE

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ↊ | ↋ | 10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ↊ | ↋ | 10 | 11 | 1 |
| | 4 | 5 | 6 | 7 | 8 | 9 | ↊ | ↋ | 10 | 11 | 12 | 2 |
| | | 6 | 7 | 8 | 9 | ↊ | ↋ | 10 | 11 | 12 | 13 | 3 |
| | | | 8 | 9 | ↊ | ↋ | 10 | 11 | 12 | 13 | 14 | 4 |
| | | | | ↊ | ↋ | 10 | 11 | 12 | 13 | 14 | 15 | 5 |
| | | | | | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 6 |
| | | | | | | 12 | 13 | 14 | 15 | 16 | 17 | 7 |
| | | | | | | | 14 | 15 | 16 | 17 | 18 | 8 |
| | | | | | | | | 16 | 17 | 18 | 19 | 9 |
| | | | | | | | | | 18 | 19 | 1↊ | ↊ |
| | | | | | | | | | | 1↊ | 1↋ | ↋ |
| | | | | | | | | | | | 20 | 10 |

## DOZENS MULTIPLICATION TABLE

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ↊ | ↋ | 10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 6 | 8 | ↊ | 10 | 12 | 14 | 16 | 18 | 1↊ | 20 | 2 |
| | 9 | 10 | 13 | 16 | 19 | 20 | 23 | 26 | 29 | 30 | 3 |
| | | 14 | 18 | 20 | 24 | 28 | 30 | 34 | 38 | 40 | 4 |
| | | | 21 | 26 | 2↋ | 34 | 39 | 42 | 47 | 50 | 5 |
| | | | | 30 | 36 | 40 | 46 | 50 | 56 | 60 | 6 |
| | | | | | 41 | 48 | 53 | 5↊ | 65 | 70 | 7 |
| | | | | | | 54 | 60 | 68 | 74 | 80 | 8 |
| | | | | | | | 69 | 76 | 83 | 90 | 9 |
| | | | | | | | | 84 | 92 | ↊0 | ↊ |
| | | | | | | | | | ↊1 | ↋0 | ↋ |
| | | | | | | | | | | 100 | 10 |

Published by

THE DUODECIMAL SOCIETY OF GREAT BRITAIN,

58 Scotby Village, Scotby, Carlisle, Cumb.
and
155 Leighton Avenue, Leigh-on-Sea, Essex.